Chaos

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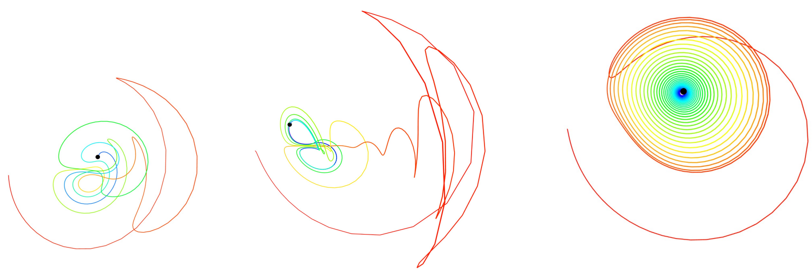

1 Introduction

Please view the front page of my website to view the *Lorenz Strange Attractor* I programmed, here is the link: arikatz.net. With every page refresh, it chooses new initial conditions for x, y, z.

Mechanics is one of the most successful theories in applied physics. Engineers are able to take the principles of Newton's Laws and employ them to design machines that perform tasks that the strength and agility of a human being simply wouldn't be able to accommodate for. Or, simply give great predictions for the events of the future. The greatest boast of the physicist is the fidelity result, and classical mechanics has been able to prove itself time and time again as effective for physics problems in the non-relativistic, non-quantum domain.

Yet, it isn't all sugar and spice. Newton's Laws give us plenty of linear results for most scenarios, which is where our most confident solutions are found as $t \Rightarrow \infty$; even without perfect measurements. However, when we obtain an expression that is non-linear, our prediction capability over long periods of time becomes incredibly dependent by the smallest precision in our initial conditions. Even the most seemingly basic apparatus is subject to this fatal flaw. Why? It isn't an issue with Newton's Laws, rather it is a mathematical consequence of *chaos*.

2 Linear vs. Non-linear Dynamics

While problems such as dropping a ball from a specific height are as simple as

$$mgh = \frac{mv^2}{2}.$$

Problems that include multiple derivatives that are codependent upon one another in a non-linear relationship (ei: not a line), chaos can ensue. The prime example of such phenomena is the *Lorenz Strange Attractor*:

$$dx/dt = \sigma(y - x)$$

$$dy/dt = x(\rho - z) - y$$

$$dz/dt = xy - \beta z$$

While σ, ρ, β are constants that mostly determine the general shape of the curve, the initial x, y, z position will have a profound effect on where we can expect our curve to be over longer periods of time. In fact, while $x = 0$ and $x = 0.0001$ seem to be nearly indistinguishable, the location of our particle takes infinitely disparate paths over longer periods of time. While these paths may all consist in a general domain, predicting where in the domain our particle will be is dependent on extreme precision of our initial condition. For instance, a double pendulum takes on a non-linear form, breeding chaos as well. It's not that these systems are not deterministic, or random; rather, it is that nearby solutions to these systems with slightly offset parameters of these equations are not necessarily within proximity. Solution to differential equations like these require infinite precision, which experimentally isn't possible. Therefore, we are limited to try our best to predict the short term instead. This is why the weather report will almost always lose its reliability past 10 days: it is one of the most chaotic systems we care to study; while we generalize jet streams and large convection currents into our calculations, there is no telling how smaller events in the atmosphere contribute to the future weather in the long term. Observe how the tails in red in demo images at the top of this paper above I marked the time of location relative to the start by color (red being earliest) start with the same u-shape. However, as time goes on they seem to diverge from each other's position indefinitely. This embodies why the weather report is solid for a week, but loses merit over months.

3 Analysis of Strange Attractor

On my website, the attractor displayed is the *Lorenz System*, founded by Edward Lorenz to model basic convection currents. The particle doesn't actually represent the position of a molecule in fluid flow, rather it describes x = intensitty of convection motion, y = temperature difference between the ascending and descending currents, and z = distortion of the vertical temperature profile.

By setting our derivatives to be the components of a vector field $F(x, y, z)$, we can study the general theme of various paths in our vector field.

$$\mathbf{F}(x, y, z) = \sigma(y - x)\hat{\imath} + (x(\rho - z) - y)\hat{\jmath} + (xy - \beta z)\hat{k}$$

,

$$\text{div}(\mathbf{F}) = -\sigma - 1 - \beta$$

$$\text{curl}(\mathbf{F}) = 2x\hat{\imath} - y\hat{\jmath} - (z - \sigma)\hat{k}$$

Given that $\sigma + \beta > 1$, the divergence will be negative. This means there will be a compression across the entire vector field system. After all, this means the system is really only an "attractor" for the σ, β condition. Also, considering it has a net curl other than the zero vector, energetically this vector field is **not** conservative. After all, simply changing the starting point drastically changes the route taken.

4 Closing Thoughts

While Newtonian mechanics provides determinism for problems theoretically, we must also consider the non-linear effects of chaos when using imperfect physical measurements to simulate the stages of a physical system, experimentally. As I continue to learn physics, I hope to take my knowledge of chaos with me to identify the areas of physics that are not hard to predict to due a lack of theoretical understanding, but rather an inherent deficiency in the precision of experimentation.